Entry and Exit Strategies in Migration Dynamics

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Abstract

In this paper, we look at the role of combined entry and exit strategies in migration. This develops a real option model in which the immigrant community in a host country is described as a club and the immigrant's benefits are an inverse U-shaped function, which depends on the size of the district. By using this framework, the paper's aim consists in trying to study in depth migrants' behaviour and in particular their duration in the host country. There are two threshold levels; the first triggers the migration choice, while the second triggers the return to the country of origin. The difference between the two thresholds defines a region of inaction (hysteresis) i.e., the length of the immigrant's stay in the host country (duration). The theoretical results show that: a) the phenomenon of hysteresis is amplified by communities both in entry and in the exit cases; b) migration policies that try to exacerbate entry, might increase duration of immigrants and increase the migration stock in the host country. Furthermore, the community could reduce the minimum wage level required to trigger both exit and entry. This fact could explain why we sometimes see migration inflows with a low wage differential and also with underemployment. Finally, we look at some possible further extensions: by introducing heterogeneity among immigrants, it could explain migration inflow for different skill levels and it also shows some theoretical implementations of including policy shocks in the migrant's choice.

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Keywords: Migration, Real Option, Theory of Clubs, Network Effect.

1 Introduction

So far, theoretical approaches that use real option frameworks to study migration choice, assume that migration is an irreversible choice. Nevertheless, economic literature has shown that migration could also be thought as a temporary phenomenon (Hill, 1987; Djajic, 1989; Dustmann, 2001; Dustmann and Weiss, 2007). Therefore, real option approaches should be extended by considering migration as the combined effect of entry and exit strategies. Developing this idea, we have included combined entry and exit strategies applied to migration study. Moreover, the paper looks at how the immigrants' community, described as a club, can affect migration net waves in the host country. What happens in the labour market? How does it change migration choice?

Much economic research considers migration as permanent[1]. The fact that the migration decision is in many cases at least partially irreversible, added to uncertainty over wage differentials and the economic conditions in the host country[2] is an important element that enabled Burda (1995) to adapt the real option approach to migration decisions. This choice has been assumed, in line with Sjaastad (1962), as an investment decision. Burda's results show that individuals prefer to wait before migrating, even if the present value of the wage differential is positive, because of the uncertainty and the sunk costs associated with migration [3]. Therefore, the novelty introduced by the real option approach consists in studying the dynamic choice of a representative agent, taking into account the value of postponable choice, i.e., the value of waiting.

Subsequently, Khwaja (2002), Anam *et al.* (2007), Moretto and Vergalli (2008), developed Burda's approach by describing the role of uncertainty in the migration decision. Another study that used real option for migration is Feist (1998). Here, he analysed the option value of the low-skilled workers to escape to the unofficial sector if welfare benefits are too close to the net wage in the official sector. In a recent work, Vergalli (2008) studied migration choice by merging the real option approach of investment decisions and work on the classical theory of clubs in a unified framework. Here, he studied the role of community in migration dynamics.

Nevertheless, migration is also studied as a temporary phenomenon. Some theoretical papers have supported this approach. Hill (1987), developed a life-cycle model of immigrant behaviour to determine total time allocated to home-country and foreign-country residence and the number of migratory trips. In his paper, Hill had two important results: a) on the one hand, lifetime participation in the foreign labour market would be more sensitive to changes in the home wage than to equal but opposite, changes in foreign wages. This fact means that policies to control migration flows are more effective in the country of origin than in the host country ; b) on the other hand Hill finds that changes in travelling costs have predictable effects on the number of border crossing but not for the total time spent in the foreign labor market. Djajic and Milbourne (1988), explicitly stressed the importance of considering migration as a temporary phenomenon and developed a life-cycle model to study the effect of wage differentials in determining migration flows and their final effect on equilibrium wages.

Djajic (1989) also examined the behaviour of utility-maximizing migrants in a system of guest-worker migration. This paper stressed the difference between permanent and temporary migration. While a permanent migrant is primarily interested in the real-wage differential between countries of immigration and emigration, a guest worker's decision depends on both real and nominal differentials. The relative importance of the nominal differential is found to be inversely related to the degree of concavity of his instantaneous utility function. Berninghaus and Seifert-Vogt (1988), developed a model in which each guest worker plans to accumulate capital in the host country (hypothesis supported also by Piore, 1979) for investment in the home country after return migration. However, due to incomplete information about economic variables of the host country and in the home country, each immigrant might prolong his stay. In other words, temporary migration might turn into permanent migration.

Stark (1992), used the theory of relative deprivation and risk spreading to explain why migrants may return to a less rich economy or region. Dustmann (1995, 1997) showed that "further motives for a return migration are a high purchasing power of the host country currency in the migrant's home economy, and higher returns to human capital, accumulated in the host country, in the home economy".

Moreover, some empirical estimates support the temporary approach showing that "more than two thirds of foreign workers to the Federal Republic have returned" (Böhning, 1987); or that "85% of Greeks have migrated to West Germany gradually returned" (Glytsos, 1988).

Other work about US migration, reported that between 1908 and 1957 about 15.7 million persons immigrated to the US and about 4.8 million emigrated (Jasso and Rosenzweig, 1982) or that about one third of legal immigrats to the US re-emigrated in the 1960's (Warren and Peck, 1980). In line with these results, Dustmann (2003) analysed optimal migration durations despite persistently higher wages in the host country. An important result is that if migration is temporary, the optimal migration duration may decrease even if the wage differential grows (i.e., there was an inverse U-shaped relationship between durations and wages). This is due to the higher weight of marginal utility of wealth (income effect) than the marginal value of staying in the host country (relative wage effect). This result was also verified when each immigrant must simultaneously decide about the optimal migration duration and their after-return activity (Dustmann, 2002).

Since temporary migration is an important feature of the migration phenomenon, some papers have tried to study the role of migration policies in controlling inflow and outflow. Faini (1996) emphasized how "allowing for the effect of migration controls on the return migration can provide the key to understanding, (1) why return policies were relatively ineffective and (2) why return propensities declined after 1974 despite the increase in the host countries' employment".

Magris and Russo (2005), also showed that there is a trade-off between frontier closure and migration duration. In particular, they showed that strict regulation of entry decreased both inflow and outflow, and its net impact on the number of foreign residents is undetermined.

In a recent paper, Bijwaard (2009) analysed demographic factors that influence the migration dynamics of immigrants in The Netherlands. By applying a mover-stayer model for the dynamic process of migration and allowing for both permanent and temporary migrants, he identified the underlaying timing determinants. His empirical results disclose differences among migrants by migration motive (among labor-migrants, family reunion migrants, family-formation migrants and students) and by country of origin. Students were the most prone to leave and family forming migrants were the least prone to leave. Moreover, migrants from countries that used to send guestworkers in the 60s and 70s of the previous century to The Netherlands, in particular Turkey and Morocco, were more often permanent than migrants from western countries.

Therefore, as Berninghaus and Seifert-Vogt (1988) proved, "temporary migration might turn into permanent migration". On the one hand, uncertainty over economic conditions in the host country as well as in the home country might prolong the optimal duration of migration (Berninghaus and Seifert-Vogt, 1990, page 205). On the other hand, during the first years in the host country, a sociocultural assimilation of migrants takes place that might change their preferences. This assimilation is accompanied by disintegration in the home country. "Consequently, temporary migration might turn into permanent migration as a result of cultural assimilation" (see also Piore, 1979).

As uncertainty might prolong the duration of stay and since incomplete information alone "suffices to induce migration flows even between countries that can be regarded as "identical" from an economic point of view" (Berninghaus and Seifert-Vogt shown, 1990, page 28), uncertainty is an important key-element that should be taken into account.

Given that uncertainty is a focal point, we therefore have all the ingredients for a real option approach in line with Burda's work but also agreeing with literature on temporary migration. We have introduced a real option model with combined entry and exit strategies. Its advantages are as follows: a) uncertainty is central; b) it develops a countinuous time model extending the benchmark two period models[4]; c) duration analysis with entry-exit strategies is as if two permanent decisions were combined into a single one. This idea may agree with the hystersis process described by migration duration.

So far we have introduced the idea behind the peculiar model adopted. Now we must clarify the main variables used in our approach. Generally in theoretical economic literature, migration choice depends on the wealth difference between the country of origin and the host country, as "*people migrate in order to increase their welfare*"[5]. Therefore, the wage differential between the host country and the country of origin is assumed to be the main variable affecting migration (Todaro, 1969; Langley, 1974; Hart, 1975; Borjas, 1990, 1994). However, it is not sufficient to totally explain migrant behaviour; as the focal role of community networks in the migrant's choice is also important (Boyd, 1989; Bauer and Zimmermann, 1997; Winters et al., 2001; Bauer et al., 2002; Coniglio, 2003; Munshi, 2001, 2003; Heitmueller, 2003, Moretti, 1999[6]).

Additionally, population size and migration are also important as a congestion effect may appear, given that the number of public services users reduces productivity gains (Braun, 1993; Krichel and Levine, 1999; Clemente *et al*, 2008). Bauer *et al.* (2002) examined the relative importance and interaction of two alternative explanations of immigrant clustering, i.e.: 1) network externalities and 2) herd behaviour. The same theme is also studied in Epstein and Gang (2004), where the authors analyse the roles that "other people" play in influencing potential migration decisions.

The moment immigrants settle in a country, they have to acquire a place in that new society. This is true not only for physical needs such as housing, but also in the social and cultural sense. Therefore, the role assumed by integration process becomes essential, as immigrants become accepted into society, both as individuals and as groups. Integration is not only taking place - as is often supposed - at the level of the *individual immigrant,* but also at the *collective level* of the immigrant group. In fact, when a immigrant enters a new society, he/she begins to build a group of people (or he/she enters a group if it is already exists), based on affinities, religions and way of life. This group is generally called "community".

The process of integration is also related to the level of *institutions*, which come in two broad types. The first are general public institutions of receiving societies or cities, such as the education system or institutional arrangements in the labour market or the dimension of the urban area in which the community develops. The second kind belongs to specific types of immigrant group themselves, such as religious or cultural institutions. This aggregate of individuals that uses, like a family, the same goods, "*deriving mutual benefit sharing [...] production costs, the members' characteristics, or a good characterised by excludable benefits*", can be modelled by following the economic theory of "clubs" (Sandler and Tschirhart, 1980; Buchanan, 1965; Berglas, 1976, Vergalli, 2008).

The role of "other people" seems to be crucial to complete wage differentials effect on migration choice in a structural model. We have therefore introduced the role of community into the entry and exit strategies of migration, trying to discover what happens to migration choice when we assume temporary migration in a real option framework.

This paper is organised as follows: section 2 presents the model and the basic assumptions. Section 3 develops the theoretical framework that combines real option theory and the network effects, namely the optimal migration strategy in the presence of positive and negative externalities and shows the main results. Section 4 shows some numerical results, while section 5 comments on the role of community in the migration dynamics. Section 6 is devoted to contextualize our results into the economic literature. Section 7 adds some further extensions and finally section 8 summarises the conclusions.

2 The Model

This section presents a continuous-time model of migration where the differential benefits of migration including wage differential, evolves in a stochastic manner over time with ongoing uncertainty.

It is possible to summarise the main assumptions:

1. There are two countries: the country of origin where each potential migrant takes his decision and the host country.

2. At any time t, each individual is free to decide to migrate to the new country. Individuals discount the future benefits at the interest rate ρ.

3. All immigrants are identical, live infinitely, or choose vicariously for their descendants who will remain in the receiving country forever[7]. Their size dn is infinitesimally small with respect to the total number of inhabitants[8].

4. Each individual enters a new country undertaking a single irreversible investment which requires an initial sunk cost K. If he/she wants to return to his country, he/she must pay another sunk cost, called E[9].

5. The migrant faces some known constant variable costs of operation, called C[10]. This cost might include legally required termination payments for houses, the buildings of the community he decided to sustain, the costs for buying a return ticket to his country and the loss of some business underway.

6. The wage differential for each migrant, called x, follows a geometric diffusion process [11]:

$$dx = \alpha x dt + \sigma x dw \tag{1}$$

   with $x_0 = x$ and $\alpha, \sigma > 0$. The component dw is a Weiner disturbance defined as $dw(t) = \varepsilon(t)\sqrt{dt}$, where $\varepsilon(t) \sim N(0,1)$ is a white noise stochastic process (see Cox and Miller, 1965). The Weiner component dw is therefore normally distributed with zero expected value and variance equal to: $dw \sim N(0, dt)$. From these assumptions and from (1) we know that $E[dw] = 0$; $E[dx] = \alpha x dt$.

7. In the host country there is a community of ethnically homogeneous individuals. Each individual instantaneously becomes a member (finding a job) when he enters the host country[12].

8. The community net benefit function for each member is an inverse U-shaped with regards to the number of members and can be modeled by using the "theory of clubs" as in Vergalli (2008) and also Bauer et al. (2002). Formally, in a given instant t, the migrant's utility function can be reduced to [13]:

$$U(x, n) = x + \theta u(n) \tag{2}$$

where θ is a scale factor. The function $u(n)$ is twice continuously differentiable in n, and increasing over the interval $[0, \overline{n})$ and decreasing thereafter. The assumption of an inverse U-shaped benefit function combines network effect (in line with Moretti, 1999; Munshi, 2003) and congestion and competition effects in the host country (Bauer et al., 2002; Clemente *et al.*, 2008). The benefit function is separable in x and $u(n)$.

3 Results

To solve the optimal decision problem of a potential immigrant or emigrant, we use standard real option approach as showed in Dixit-Pindyck (1994, pp. 216-218) or in Dixit (1989, page 625). We must distinguish between the value of staying idle (V_0) and the value of belonging to the host country, (V_1).

Let us start with the idle entrant: the resulting equation for $V_0(x)$ is:

$$\frac{1}{2}\sigma^2 x^2 V_0''(x) + \alpha x V_0'(x) - \rho V_0(x) = 0 \tag{3}$$

where $V_0''(x) \equiv \partial V_0 / \partial x$.

Its general solution:

$$V_0(x) = A_1 x^{\beta_1} \tag{4}$$

where A_1 is a constant to be determined, and $\beta_1 > 1$ is a known constant whose value depends on the parameters ρ, σ and α. This value is valid in the interval in which each individual remains at home. Let us call x_h the threshold level of wage differential that triggers entry. Therefore, (18) is valid over the interval $(0, x_H)$.

Now, let us consider the value of living in the host country for the migrant:

$$\frac{1}{2}\sigma^2 x^2 V_1''(x, n) + \alpha x V_1'(x, n) - \rho V_1(x, n) + x - C + \theta u(n) = 0 \tag{5}$$

The general solution of this equation is:

$$V_1(x,n) = B_2 x^{\beta_2} + \frac{x}{\rho - \alpha} + \frac{\theta u(n) - C}{\rho} \tag{6}$$

where the last two terms are the value of remaining in the country despite any losses and the first two terms are the value of the option to abandon the country.

If (19) is the value of stay in the host country, it must be valid over a given threshold x_L that triggers entry. Therefore, (19) is valid for x in the range (x_L, ∞)[14].

Equations (18) and (19) can be solved by using ordinary value-matching and smooth-pasting conditions showed in the appendix (equations (20), (21) and (22), (23) respectively). Therefore, as far as real option framework is concerned, we obtain two threshold wages: an upper bound (x_H) and a lower bound (x_L):

- if the threshold x rises above the upper bound x_H, then the immigrant enters the host country;
- if the threshold is below x_L, then the migrant returns to his country of origin;
- in between, in the interval $(x_L,\ x_H)$, each individual remains idle in the host country (if he has already emigrated) or in his country of origin (if he is still in his own country). This kind of inaction is a hysteresis process that magnifies and partially explains the duration of migration[15].

To better analyse the influences of the immigrants'community on migration flow and in particular its duration, we have defined the following function, by using the solutions (17) and (5):

$$\begin{aligned} G(x,n) &= V_1(x,n) - V_0(x) = \\ &= -A_1 x^{\beta_1} + B_2 x^{\beta_2} + \frac{x}{\rho - \alpha} + \frac{\theta u(n) - C}{\rho} \end{aligned} \tag{7}$$

where $G(x)$ represents on the interval (w_L, w_H) the *migrant's incremental value of migrating.* If we do not take into account the presence of the community, equation (30) becomes:

$$\begin{aligned} G(x) &= V_1(x) - V_0(x) = \\ &= -A_1 x^{\beta_1} + B_2 x^{\beta_2} + \frac{x}{\rho - \alpha} - \frac{C}{\rho} \end{aligned} \tag{8}$$

The general shape of $G(x,n)$ with community, obtained by (7) - blue line - and the particular case without community benefit ($G(x)$), (8) - dotted line, are shown in figure 1.

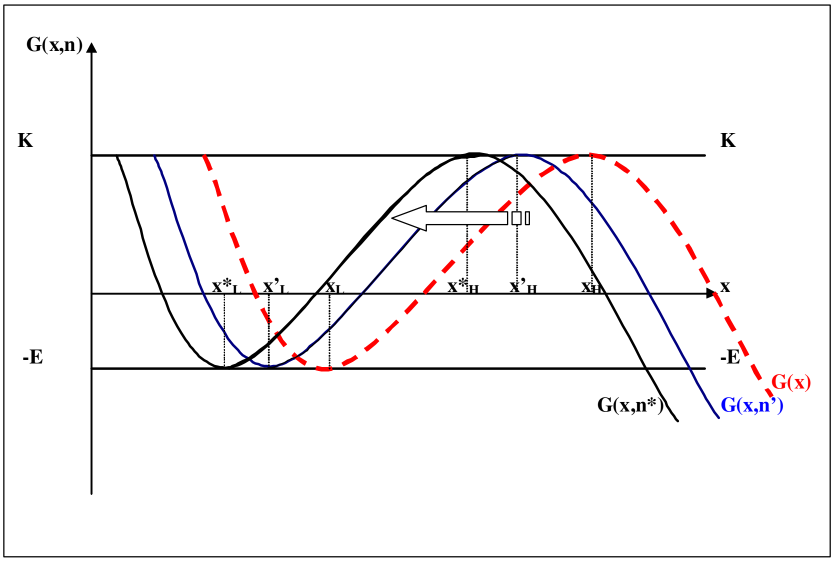

Figure 1: Entry and Exit Strategies

The $G(x,n)$ curves show the optimal threshold levels for given numbers of immigrant in the community. When the shock is between the upper (x_H) and the lower (x_L) level nothing happens and each immigrant remains where they are. This happens both without a community (red-dotted line) and with a community (continuous line). Let us point out that when the shock that moves according to (1), touches or crosses (on the abscissa) the upper (lower) level, there is a new entry (exit) of immigrants into the host country. With a community, this implies a change in the total number of members and the $G(x,n)$ function moves accordingly. There is an optimal level of individuals $\overline{n}$ that defines the optimal dimension of community in which the benefit is at the maximum (Vergalli, 2008; Moretto and Vergalli, 2008).

So, if the community size is lower than the optimal level, then each individual increases the benefit to migrate due to network effects. For this, a new entry moves the $G(x,n)$ curve to the left, changing the entry-exit thresholds until the optimal dimension $\overline{n}$ is reached. After this point, the benefit added by each new member of the community decreases until marginal benefits are equal to zero. This is due to the combination of congestion and competition effects that decrease the marginal benefit of each new member. This effect moves the G(x,n) curves to the right, until the network and congestion effects are offset.

Given the focal role of the two triggers (x_H and x_L), we can calculate them by using real option methodology (see the appendix). To solve the problem for both cases (equation (7) and (8)), we must adjust A_1 and B_2 until $G(x)$ (or $G(x,n)$) becomes tangent to the horizontal lines $-E$ and K (in figure 1), and the respective points of tangency define x_L and x_H. Technically speaking, this tangency comes from the value-matching ((20) and (22)) and smooth pasting conditions ((21) and (23)) that imply that the entry-exit triggers must be optimal when the migrant's incremental value of migrating is equal to the entry-exit costs. In the general case, the optimal thresholds are:

$$x_H > C + \rho K \equiv C_H \quad (9)$$

$$x_L < C - \rho E \equiv C_L \quad (10)$$

where C_H and C_L are the Marshallian triggers[16] for entry and exit, respectively.

Note that the optimal triggers are respectively greater and lower than the Marshallian cases. This implies that the uncertainty widens the Marshallian range of inaction. Let us comment on (9) and (10): as both K and E tend to zero, both x_H and x_L tend to the common limit C. Thus, sunk costs are essential to hysteresis and therefore for the migration duration. Moreover, if either K and E tend to zero while the other remains positive, both inequalities (9) and (10) remain strict. Indeed, even if E is nil, the exit threshold x_L stays below C. Each immigrant knows that if he/she remains in the host country, he/she can avoid incurring K to reentry in future if the differential wage increases again. He/she prefers to incur some current loss in order to preserve this option.

In the case with community, the threshold levels become:

$$x_H^* > C + \rho K - \theta u(n) \equiv C_H - \theta u(n) \equiv C'_H(n) \quad (11)$$

$$x_L^* < C - \rho E - \theta u(n) \equiv C_L - \theta u(n) \equiv C'_L(n) \quad (12)$$

Note that, given that by definition $u(n) \geq 0$, in both cases $x_H \geq x_H^*$ and $x_L \geq x_L^*$. Therefore, the effect of community in migration duration is to reduce the entry-exit thresholds and thus it speeds up entry and delays exit of immigrants.

Following Dixit (1989), we can say that, if $E > C/\rho$, then the immigrant will never go home. Nevertheless, x_H does not tend towards infinity.There is a finite differential wage that entails a high value of migration to the host country, impossible to avoid. So, the exit option must be worthless. Therefore, B_2 of equation (19) must be imposed equal to zero. We therefore obtain:

$$x_H = \left[\frac{\rho - \alpha}{\rho}\right]\left[\frac{\beta_1}{\beta_1 - 1}\right] C_H \tag{13}$$

or, in the community case:

$$x_H^* = \left[\frac{\rho - \alpha}{\rho}\right]\left[\frac{\beta_1}{\beta_1 - 1}\right] C_H\left(n\right) \tag{14}$$

Moreover, if K tends towards infinity, the entry option becomes worthless and A_1 is nil. We therefore have

$$x_L = \left[\frac{\rho - \alpha}{\rho}\right]\left[\frac{\beta_2}{\beta_2 - 1}\right] C_L \tag{15}$$

or, with a community:

$$x_L^* = \left[\frac{\rho - \alpha}{\rho}\right]\left[\frac{\beta_2}{\beta_2 - 1}\right] C_L\left(n\right) \tag{16}$$

Note that when $\sigma \to 0$ this implies that $x_L^* \to C_H\left(n\right)$ and $x_L^* \to C_L\left(n\right).$ This implies that, without uncertainty, only the Marshallian zone of inaction remains.

Let us consider the comparative statics[17] of x_H and x_L with respect to C, K and E[18]. We can say: i) as C increases, both x_H and x_L increase; ii) when K increases, x_L decreases and x_H increases: that is, the hysteresis effect becomes more pronounced and this means that the duration increases. Similar effects stem from E; iii) if we keep σ at a positive level and let $K \to 0$, we have $dx_H/dK \to \infty$ and $dx_L/dK \to -\infty$. That is, when there is uncertainty, hysteresis emerges very rapidily even for small sunk costs. There are similar results when $L \to 0$.

4 Numerical Results

Here we support our theoretical results with some numerical simulations. We have assigned some values to the parameters used referring to Dixit and Pindyck (1998, page 8). Moreover, we have normalized C to 1 and we have introduced a simple $u(n)$ function to describe the threshold effect better. For the sake of simplicity, $u(n)$ is maximum in $n = 50$ and minimum in $n = 0$ and $n = 100$, as well. Let us show the optimal threshold level x_H (figure 2) and x_L

(figure 3) compared to the volatility of differential wage and the dimension of the community.

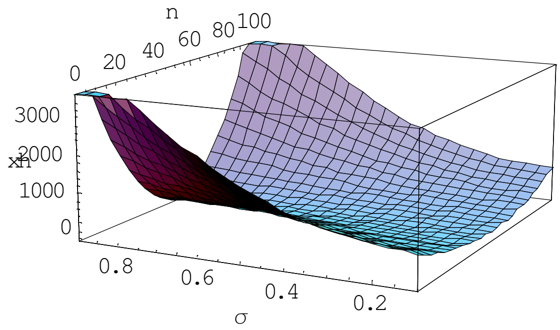

Figure 2: Optimal threshold x_H^* with respect the number of immigrants n and volatility σ.

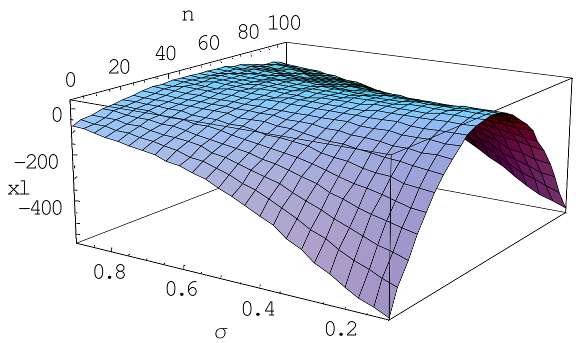

Figure 3: Optimal threshold x_L^* with respect the number of immigrants n and volatility σ.

From figure 2 and 3, we can say that in both cases, the higher the volatility, the higher the entry-exit threshold levels. Moreover, the bigger the community (for n<$\overline{n}$) the lower the optimal threshold required to migrate. These results are perfectly in line with comparative statics showed previously. However, if we analyse the combined effect of volatility and community, we can see that a high σ reduces the network effect for the lower level and increases the effect for the upper threshold. On the one hand, the volatility effect increases the threshold level required to migrate, (more uncertainty needs a higher wage differential), on the other hand, the network effect decreases the entry-exit triggers. However, when the first effect is strong, it dominates the second. The implication is that the community is unable to help each new immigrant if there are strong shocks in the labour market.

In figure 4 and 5, we show comparative statics for variable n (i.e., network effect) and C (i.e. operative costs). The results agree with theoretical economics: the community effect reduces entry and exit triggers and therefore increases the number of immigrants, while everyday costs reduce immigration flow.

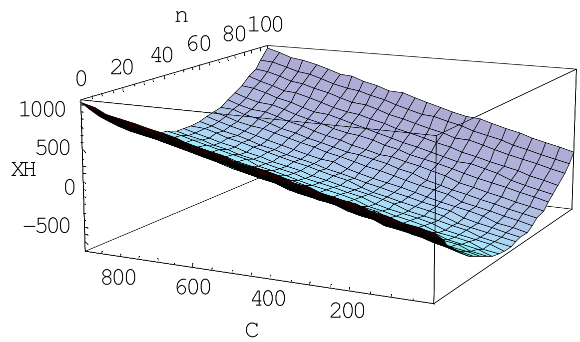

Figure 4: Optimal threshold x_H^* with respect the number of immigrants n and C.

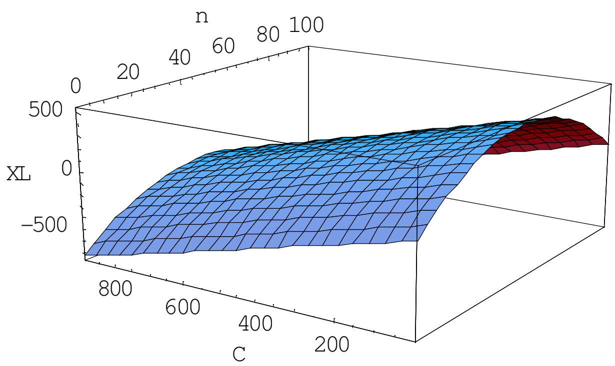

Figure 5: Optimal threshold x_L^* with respect the number of immigrants n and C.

More interesting, from a political point of view, is the effect of the entry-exit sunk costs (K and E) on the threshold levels. To show these results, we have used the comparative statics shown in appendix and the following figures 6 and 7[19]. Our focus is only on the role of entry and exit sunk costs, without taking into account the community effect showed above[20].

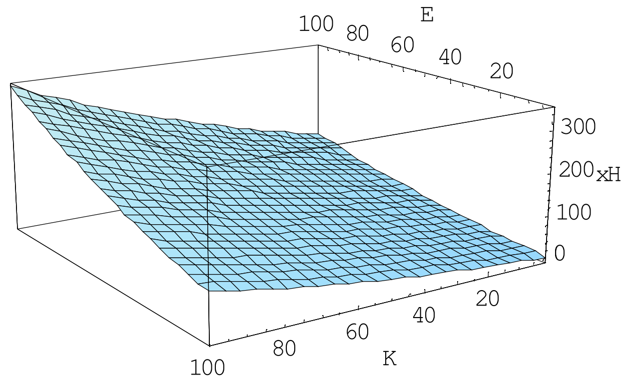

Figure 6: Optimal threshold x_H^* with respect K and E.

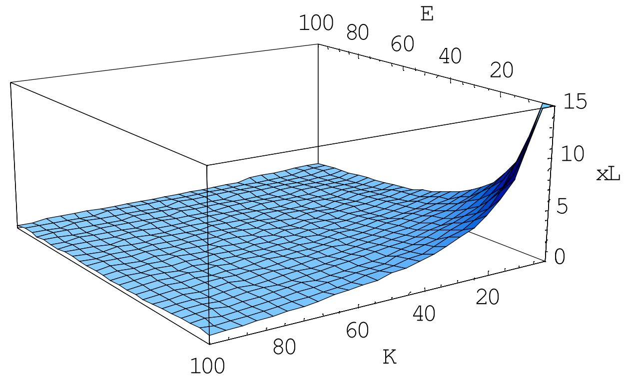

Figure 7: Optimal threshold x_L^* with respect K and E.

From figure 6 we can see that the upper threshold x_H^* arises with the entry cost K and the exit cost E. This means that if the entry cost K is high, each immigrant waits more to enter in order to avoid bad news (therefore, there is a higher option value). The second result means that each individual is more reluctant to migrate if he/she spends more cost to return home in the future. Figure 7 shows the mirror image result namely, that the abandonment threshold x_L^* falls when the entry cost K and the exit cost E rise.

Each immigrant abandons the host country with some reluctance because of his/her option value. By staying in the host country, he/she avoids incurring the entry cost once again should the wage differential process turn sufficiently favourable in the future. The net result about immigrants' duration is that, when the entry cost increases, the difference between entry-exit thresholds rises and therefore the period of stay of each immigrant increases.

This result has two implications: firstly, when a government tightens the admission requirements for each immigrants (by raising K, as in European

Union[21]) it might actually increase the duration period and therefore in the short run there might be an increased effect of lock-in for the immigration flows. Indeed, the exit flow might be locked into the host country due to a lower level of x_L^*, while the entry flows might increase for increasing shocks over the differential wage[22].

However, by looking at the mirror image of our result, we can agree with Dustmann (2003): "if migration is temporary, the optimal migration duration may decrease even if the wage differential grows larger". In our model this happens when, even if there are increasing differential wages, the entry and exit costs decrease, reducing the spread between x_H^* and x_L^*. Moreover, we must stress that while Dustmann examines the duration for only one kind of differential wage, we have analysed two thresholds, where the lowest is that which triggers exit.

In our context, if the differential wage increases it probably goes over x_L^* and when the differential wage grows larger it maintains up, avoiding that a migrant wants to return. Therefore, the Dustmann result is not unusual, but it is pefectly explained by our model.

To complete the argument, in figure 8 we show the combined effect of community and sunk costs. We have overlapped the upper (above) and lower (below) threshold levels to understand the effects of community better. As it is possible to see by looking at our figure, even if the sunk costs increase the spread between upper and lower level, this effect is mitigated by the community (when the network effect prevails against the competitive effect). On the one hand, when the network effect is high, for each given level of sunk costs, both the threshold levels decreases, thereby magnifying duration. On the other hand, the curve of the upper x_H^* is more concave than x_L^*. This means that the sunk costs effect is not balanced for x_H^* and x_L^*. Furthermore, we have the lowest spread x_H^* and x_L^*, when the network effect is at its maximum level ($\overline{n}$), *ceteris paribus.*

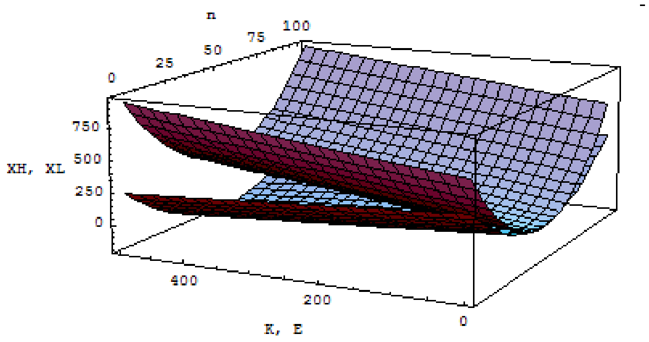

Figure 8: Optimal threshold x_H^* and x_L^* with $\rho = 0.5$ and with respect K, E and n.

5 The Role of Community in Entry-Exit Strategy

Here we have summarized the community effect for migration dynamics (entry) and duration (entry-exit difference). We have compared the entry-exit strategy without community (8) with the same strategy, with a community (7) and looked at our two previous sections (and figures 1-8). The community's effects are the following:

1. the more members in the community[23], the lower the trigger value x_H at which each individual decides to migrate (for $n < \overline{n}$). This implies that the more members for $n \in (0, \overline{n})$, the earlier the migration starts;

2. the more members in the community, the lower the trigger value of exit x_L. This magnifies the phenomenon of hysteresis to remain in the host country even if the level of the migrant's wage is low. Furthermore, the greater the benefit from the community, the lower the wage that each member needs to remain, because of a high network effect. Therefore, the community increases migration duration.

3. Comparative statics and numerical results show that the migration thresholds x_H rises and x_L falls with the investment cost K. Therefore, the larger the investment cost, the larger the option value and the greater is the reluctance to abandon. The mirror image results, namely, that the migration threshold x_H rises and x_L falls as the abandonment cost E increases. The role of community consists in modifying the effect of a

change in K and E. On the one hand the community reduces threshold levels, increasing duration (i.e. stock) and migration inflow. On the other hand, given that the concavities of the upper and lower threshold levels are different, the net result is that when the network effect is strong the x_H and x_L spread decreases, reducing hysteresis.

4. The effect of increasing uncertainty: Berninghaus and Seifert-Vogt (1990) showed that a country becomes more attractive for immigration if the "uncertainty" in the quality of life distribution,[...] increases. They also demonstrated (1987) how the result could be interpreted in the direction of decreasing information. On the contrary, the real option approach applied in our framework shows that more uncertainty increases the threshold level and therefore postpones the entry of immigrants; so, that country becomes less attractive.

   In conclusion, the benefit of community increases the number of immigrants in the host country, the duration of their stay and higher unemployment. Therefore,

Proposition "the existence of a community of immigrants in the host country magnifies the hysteresis' phenomenon. This fact explains migration inflow with high unemployment rates and low wages".

6 Results in Economic Literature

6.1 The Harris_Todaro paradox

In two seminal papers, Todaro (1969) and Harris and Todaro (1970) developed a canonical model of rural-urban migration. The main idea is quite simple as it states that migration will occur as long as the expected urban income (i.e., income times the probability to find an urban job) is higher than the rural one. These papers have been so influential that they are referred in the literature to as the Harris-Todaro model. One of the main issues was that creating urban jobs may increase rather than decrease urban unemployment because of the induced negative effect on rural migration, which may outweight the positive effect of creating jobs (Todaro, 1976). This is referred to as the Todaro paradox.

The paradox is due to the assumptions that in choosing between labour markets, risk-neutral agents consider expected wages, that the probability of obtaining urban employment is approximated by the ratio of urban jobs to the urban labor force; and that the urban wage rate is considerably and consistently higher than the rural wage rate. Under these assumptions, inter-labour market (rural-urban) equilibrium creates urban unemployment. This unemployment ensures that the expected urban wage is equal to the rural wage (which is assumed to be constant throughout). The repercussion of this simple set of assumptions is that unlike received wisdom, once the migration response is factored in, several policies aimed at reducing urban unemployment will actually raise urban unemployment rather than reduce it.

In the Harris-Todaro model, migration is regarded as the adjustement mechanism by which workers allocate themselves between different labor markets, some of which are located in urban areas and some in rural areas, while attempting to maximize their expected income. The effects of this model change the magnitude and the sign of the Harris-Todaro (1970) paradox. By reducing the threshold level to migrate (i.e., the minimum wage) compared to a labour market without a community, the unemployment rate is not efficient to counterbalance migration inflow. In this case, the Todaro paradox is diluted.

This effect is similar to a reduction of "unemployment benefit" imposed by the goverment as described in Zenou (2005). Here, a Todaro paradox exists if a reduction in the urban unemployment benefit (exogenous variable and policy instrument) increases both urban employment and unemployment. This is a paradox since reducing unemployment benefit has the natural effect of increasing urban employment but the opposite effect of increasing urban unemployment.

In the case of a search-matching model where wages are bargained, a Todaro paradox may exist if a condition on parameters is satisfied. The benefit of a community has a direct positive effect on bargained wages. Because it is cheaper and therefore more profitable to hire a worker, more firms enter the urban labor market and create more jobs and so rural-urban migration increases. However, when the community benefit decreases, there is also a direct negative effect on migration since urban wages are lower and so less rural workers migrate. The net effect is therefore ambiguous. A condition that guarantees that the indirect positive effect on migration is larger than the direct negative effect leads to a Todaro paradox since a reduction in community benefit increases both urban employment and unemployment.

6.2 Effects of community in countries with centralised wage-setting and no labour mobility

By looking at figure 9, another comment should be made for a two-countries centralised wage-setting framework (Boeri and Brücker, 2005).

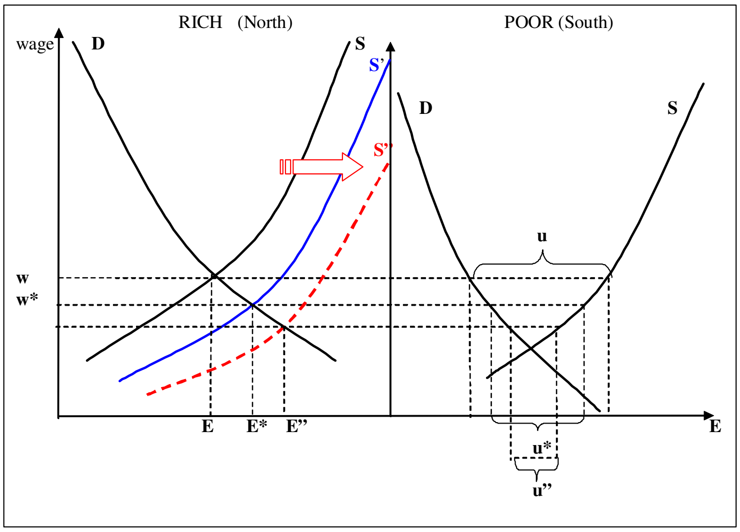

Figure 9: Effects of community in countries with centralised wage-setting and no labour mobility.

With wage compressing institutions, international migration can reduce unemployment also in low-productivity (high-unemployment) regions. This additional "greasing the wheels" effect of migration is visually shown in the above diagram. The panel on the left-hand side shows the market-clearing wage prevailing in the dynamic regions (called here the Rich or North) which is also paid – due to the imposition of the same contractual minimum throughout the country – in the Poor or South. At the initial equilibrium, the South experiences unemployment as the Northern wage acts as a binding minimum wage.

Migration has two useful functions in this context. On the one hand, it increases employment and reduces wages in the North by shifting labour supply to the right (as shown by the blue line, $S\prime$). On the other hand, by acting on Northern wages, migration reduces labour costs also in the South (from w to w^*) partially absorbing its unemployment (which shrinks from u to u^*). As seen above, the community reduces the entry-exit threshold, i.e., the centralised minimum wage. Therefore, the effect is an increase in the supply to the red dotted line, by increasing the number of immigrants. The consequence is a rise of employment (E") in the rich region and a reduction of the unemployment rate in the poor region (u").[24]

7 Some extensions

So far we have studied the role of the community in the entry and exit of a migrant, developing a model similar to Burda and implementing an extension taken from the theory of clubs. We have observed that a homogeneous community of individuals reduces the trigger level at which the individual decides to migrate and also reduces the wage level at which each migrant wants to go back. We have assumed that all individuals are homogeneous and we have not take into account any policy choices. But what happens if the individuals are heterogenous? Here, we can generalise the model in a simple manner, considering two possible applications.

Taking into account the possibility of different skills among migrants, we could assume that they can gain higher wages the higher their skill level. That is, we could simply assume that the value of migrating in the host country is:

$$V(x,n) = E_0 \left[\int_0^\infty (\Psi_i x + \theta u(n)) e^{-\rho t} \left| n_0 = n, \theta_0 = \theta, x_0 = x \right| \right]$$

with $\Psi_i > \Psi_j$[25], if the skill level i is greater than j.

By this function and following the previous method, we can demonstrate that the first inflow would be composed of high skill, because of a greater benefit for the same shock x. Furthermore and for the same reason they would remain longer. Nevertheless, increasing the community's members, the benefit should increase, *ceteris paribus*. This should mean a reduction of the threshold level and the entry also of low-skill immigrants. A possible policy for selecting the migrants'skills could consist in increasing entry costs, as Urrutia (2001) suggests. However, as we have seen, this option would increase the hysteresis phenomenon of remaining in the community. Furthermore, the policy makers generally help the integration of new groups because of their lower possibilities. Although this policy choice is right for ethical reasons, it would not only stimulate migration, increasing the phenomenon of hysteresis, but also reduces the average level of migrants' skills.

Governments can not only use measures to reduce uncertainty facing potential investors, they can create uncertainty through the prospect of policy change.

This feature is relevant in migration analysis because a new law could increase or reduce the costs of integration for all immigrants. It is commonly believed that expectations of policy shifts can have powerful effects on investment decisions. We show a possible analytical implementation in appendix D. An additional insight about heterogeneous individuals is the assumption that different migrants bear diffent sunk costs (see Moretto and Vergalli, 2008; Bijwaard, 2009).

For example, let us assume that there are four groups of labor-migrants, family reunion migrants, family-formation migrants and students. They probably have different social relationships: e.g., family-formation migrants have closer links than students. This implies that the stronger the ties among individuals, the higher the sunk cost to return home E (or to migrate, K) must be. Given that the trigger level decreases with E, individuals delay their return home. Therefore, this could support Bijwaard's result, according to which "students are the most prone to leave and family formation migrants are the least prone to leave".

8 Conclusions

This study analyses the role of combined entry and exit strategies in the migration process. It develops a real option model in which the community of immigrants in the host country is described as a club and the immigrant's benefits is a U-shaped function, depending on the dimension of the district. In particular, this paper applies some extensions taken from Dixit and Pindyck (1993, pp. 217-222) regarding the combined entry and exit strategies of migrants. There is a threshold that triggers entry and a second that triggers the return to the country of origin. The difference between the two thresholds defines a region of inaction (hysteresis) that is the length of stay of each immigrant in the host country (duration).

Our theoretical results show that when a government worsens admission requirements for each immigrants, it might increase the duration period and therefore in the short run there might be an increased effect of lock-in for the immigration flows. Indeed, the exit flow might be locked into the host country due to a lower optimal exit wage (x_L^*), while the entry flows might increase for increasing shocks over the differential wage. Moreover we can also explain why sometimes the optimal migration duration "may decrease even if the wage differential grows larger" Dustmann (2003).

In our model this happens when, even if there are increasing differential wages, the entry and exit costs decrease, reducing the spread between optimal entry (x_H^*) and exit (x_L^*) differential wages. Furthermore, we show that hysteresis is amplified by the existence of a community both in entry and in exit. The community can reduce the minimum wage level required to trigger both exit and entry. This could explain why in some cases there is migration inflow with a low differential wage and also with underemployment as previously shown by Todaro (1970).

This result has some theoretical implications: in a framework with centralised wage-setting and no labour mobility (Boeri and Brücker, 2005), the consequence is a rise of employment in rich regions and a reduction in poor regions, due to reduced minimum wages and increased labour supply. Finally, by adding heterogeneity among immigrants, we show that the skilled imigrants, should be the first wave to migrate because of the higher expected wage. This increases the community and reduces the threshold level, thus incentivating entry of low skilled immigrants.

Notes

[1]In this respect, see Chiswick (1978, 1980), Borjas (1985), Bell (1997), Friedberg (2000), Barth et. al (2004).

[2]See Berninghaus and Seifert-Vogt (1988) and Piore (1979).

[3]Investment is defined as the act of incurring an immediate cost in the expectation of future payoff. However, when the immediate cost is sunk (at least partially) and there is uncertainty over future rewards, the timing of the investment decision becomes crucial (Dixit and Pindyck, 1994, p.3).

[4]See for example Berninghaus and Seifert-Vogt (1988, 1990), Karayalcin (1994), Dustmann (2002, 2003, 2004).

[5]Khwaja, Y., "Should I Stay or Should I Go? Migration Under Uncertainty: A Real Option Approach", mimeo, March, 2002

[6]For example, in Moretti (1999) both the timing and the destination of migration could be explained by social networks in the host country.

[7]It is possible to show that the "*sudden death*" formulation is a very natural generalisation of the infinite-life case (Dixit and Pindyck, 1993, p.205).

[8]Regarding the number of migrants, it is taken to be infinitesimal in line with the standard literature on Real Options, in order to be able to use options in continuing time. In particular, Grenadier (2002),- in a paper that provides a tractable approach for deriving equilibrium investment strategies in a continuous-time Cournot-Nash framework - showed that the assumption of infinitesimal increments in the variable n is a generalization of the case developed in discrete time and shows the same qualitative results. We can also refer to Bartolini (1993, 1995) who studied the effect of a limit to the number of firms (assumed infinitely small) in a given market. Bartolini's model can also be used to find the trigger point here and in Vergalli and Moretto (2008) and Vergalli (2008).

[9]So, we explain that temporary migration is assumed but as the combination of two actions: entry and exit. It is assumed that both choices must bear sunk costs. Therefore, we implicitly suppose that each individuals must choose between two irreversible decisions. This theoretical novelty is in line with the hysteresis process in temporary migration showed in the economic literature.

[10]This could represent the costs of integration.

[11]This assumption is standard in literature concerning real options approach to describe migration. See, Burda (1995), Khwaja (2002), Anam et. al. (2007), Moretto and Vergalli (2008), Vergalli (2008). The wage differential is described by equation (1) in the text. It is composed of two elements: the first part ($\alpha x dt$) is not stochastic, and describes how the wage differential varies with time (in this case, the drift αx is greater than 1 if the host country's growth is greater than country 2 and is between zero and one if there is a convergence between the two countries) and a second stochastic part ($\sigma x dw$) described a Wiener process. The latter tries to capture the randomness of the wage differential that tends to arise from two

sources: the labour market in the host country and the labour market in the country of origin. The combination of the sources of uncertainty tends to define the stochastic part of the wage differential. It is also possible to use an Osteirn-Olemberg process that has a Wiener component and moves around a given mean value. This model could be adopted when the differential wage between the two countries remains constant or tends to increase smoothly. But this is a special case of geometric brownian motion and does not provide a solution in closed form, neither does it improve the qualitative results in our model. Therefore we adopted the geometric Brownian motion.

[12]This hypothesis assumes that each immigrant is able to find a job instantly when he/she enters the host country. Moreover, he/she is perfectly integrated in his/her homogeneous community. This assumption simplifies the mathematical analysis, but we can extend this approach by solving the model in a two-stage backward model, as in Moretto and Vergalli (2008). In any case, the qualitative results do not change. See Vergalli (2008) for further details.

[13]Formally, in line with "theory of clubs" literature, we assume that there are common public goods J, such as churches, cultural centres and houses, belonging to a group of homogeneous individuals. Given that J has certain rigidities, the analysis assumes that these public goods are fixed in a given instant t or in the short run. This variable could change in the long run, until an optimal threshold J^*. In any case, for a fixed level of J or at the optimal ceiling, the benefit function is an inverse U-shaped function for the number of immigrants belonging to a community. Furthermore, if the initial level of public goods is not the optimal one, when J increases the maximum in $\overline{n}$ (i.e. $u(\overline{n})$) increases. For this, see Buchanan (1975), Berglas (1976), Cornes and Sandler (1986) or Vergalli (2008) for further details.

[14]In Appendix A, we explain how it is possible to find the parameter values of these equations, following the Dixit and Pindyck methodology. For details, see Vergalli (2005).

[15]A peculiar *caveat* must be added about the comparison between "hysteresis" and "migration duration". Hysteresis is defined (Dixit, 1989, page 622) as "the failure of an effect to reverse itself as its underlying cause is reversed. For example, the foreign firms that entered the U.S. market when the dollar appreciated did not exit when the dollar fell back to its original level". This definition is often used in industrial economics. Migration duration is in line with Dustmann (1996, 2002, 2003, 2007) and it is immigrant's lenght of stay in the host country. It is easy to understand that by definition, the migration duration strongly depends on an exit decision of each immigrant. Therefore, an hysteresis process increases the immigrants' duration.

[16]The Marshallian trigger is "the point at which the present value of the benefit exceed the cost of migration", see Anam et al. (2007).

[17]These comparative statics happen in both the cases, with or without a community.

[18]See appendix.

[19]Our figures are obtained by using the mathematical simplifications of Dixit (1991) and using Mathematica software. The same results are obtained numerically and explained in the appendix.

[20]The qualitative results will not change with community.

[21]About this, see Boeri and Brücker (2005).

[22]This insight supports also Magris and Russo (2005).

[23]for a given dimension of J.

[24]An important "*caveat*" concerning the minimum wage effect is that in this section we have tried to extend Boeri and Brücker's framework by analysing what could be the effect of the community on unemployment. For the sake of simplicity, we have assumed that all individuals are homogeneous. Indeed, this effect is general and can be applied both for blue collar and white collar immigrants. We could analyse the distinction between the two categories but the qualitative results will not change. In any case, we have extended our model in the following section 7, by taking into account what happens when immigrants are heterogeneous.

[25]Without loss of generality, Ψ can be distributed as $\Psi \sim N(\overline{\Psi}, \sigma_{\Psi})$.

A Appendix: Entry and exit strategies

Let us start with the idle entrant. The resulting equation is a differential equation for $V_0(x)$:

$$\frac{1}{2}\sigma^2 x^2 V_0^{"}(x) + \alpha x V_0^{'}(x) - \rho V_0(x) = 0 \tag{17}$$

This has the general solution:

$$V_0(x) = A_1 x^{\beta_1} + A_2 x^{\beta_2}$$

where A_1 and A_2 are constants to be determined, β_1 and β_2 are the roots:

$$\beta_1 = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left[\frac{\alpha}{\sigma^2} - \frac{1}{2}\right]^2 + 2\frac{\rho}{\sigma^2}} > 1$$
$$\beta_2 = \frac{1}{2} - \frac{\alpha}{\sigma^2} - \sqrt{\left[\frac{\alpha}{\sigma^2} - \frac{1}{2}\right]^2 + 2\frac{\rho}{\sigma^2}} < 0$$

We know that the coefficient A_2, corresponding to the negative root β_2, must be zero. Indeed, for x$\rightarrow 0$ the fact that the migration value could explode must be avoided. This leaves:

$$V_0(x) = A_1 x^{\beta_1} \tag{18}$$

This value is valid over the interval $(0, x_h)$.

Let us consider the value of living in the host country for the migrant:

$$\frac{1}{2}\sigma^2 x^2 V_1^{"}(x,n) + \alpha x V_1^{'}(x,n) - \rho V_0(x,n) + x - C + \theta u(n) = 0$$

The general solution of this equation is:

$$V_1(x,n) = B_1 x^{\beta_1} + B_2 x^{\beta_2} + \frac{x}{\rho - \alpha} + \frac{\theta u(n) - C}{\rho}$$

where the last three terms are the value of remaining in the country despite any losses and the first two terms are the value of the option to abandon the country. Because the abandonment option value should tend to zero as x becomes large, the coefficient B_1 corresponding to the positive root β_1 should be zero. This leaves:

$$V_1(x,n) = B_2 x^{\beta_2} + \frac{x}{\rho - \alpha} + \frac{\theta u(n) - C}{\rho} \tag{19}$$

this is valid for x in the range (x_L, ∞).

So, following the methodology of Dixit and Pindyck, we could solve (4) and (6) using the conditions of value matching and smooth pasting:

$$V_0(x_H) = V_1(x_H) - K \tag{20}$$
$$V_0'(x_H) = V_1'(x_H) \tag{21}$$
$$V_1(x_L) = V_0(x_L) - E \tag{22}$$
$$V_1'(x_L) = V_0'(x_L) \tag{23}$$

and substituting (4) and (6), we have:

$$-A_1 x_H^{\beta_1} + B_2 x_H^{\beta_2} + \frac{x_H}{\rho - \alpha} + \frac{\theta u(n) - C}{\rho} = K \tag{24}$$
$$-\beta_1 A_1 x_H^{\beta_1 - 1} + B_2 \beta_2 x_H^{\beta_2 - 1} + \frac{1}{\rho - \alpha} = 0 \tag{25}$$
$$-A_1 x_L^{\beta_1} + B_2 x_L^{\beta_2} + \frac{x_L}{\rho - \alpha} + \frac{\theta u(n) - C}{\rho} = -E \tag{26}$$
$$-\beta_1 A_1 x_L^{\beta_1 - 1} + B_2 \beta_2 x_L^{\beta_2 - 1} + \frac{1}{\rho - \alpha} = 0 \tag{27}$$

The four equations determine the four unknown values. We can solve the system of the four equations numerically which gives the following:

$$\left(\frac{\beta_2 - \beta_1}{\beta_1}\right) B_2 x_H^{\beta_2} = \left(\frac{1 - \beta_1}{\beta_1}\right) \frac{x_H}{\rho - \alpha} + \frac{\theta u(n) - C}{\rho} - K = 0 \tag{28}$$
$$\left(\frac{\beta_1 - \beta_2}{\beta_2}\right) A_1 x_L^{\beta_1} = \left(\frac{1 - \beta_2}{\beta_2}\right) \frac{x_L}{\rho - \alpha} - \frac{\theta u(n) - C}{\rho} - E \tag{29}$$

The results are the same as equations (14) and (16) and figures 1-9. The results and numerical simulations were obtained by using Mathematica software. The thresholds levels and Mathematica code is available from the author.

B Analytical Results

Now, to analyse the effects of the community on the migration decision, we can define the following function:

$$G(x, n) = V_1(x, n) - V_0(x) =$$
$$= -A_1 x^{\beta_1} + B_2 x^{\beta_2} + \frac{x}{\rho - \alpha} + \frac{\theta u(n) - C}{\rho} \tag{30}$$

where $G(x)$ represents on the interval (w_L, w_H) the "*migrant's incremental value of migrating*". If the same function without the community is:

$$\begin{aligned} G(x) &= V_1(x) - V_0(x) = \\ &= -A_1 x^{\beta_1} + B_2 x^{\beta_2} + \frac{x}{\rho - \alpha} - \frac{C}{\rho} \end{aligned} \tag{31}$$

Working with the function G remains useful, and helps to show its dependence on the option value coefficients. Thus, we can write $G(x, A_1, B_2)$. The value-matching and smooth-pasting conditions are:

$$G(x_H, A_1, B_2) = K, \qquad G(x_L, A_1, B_2) = -E \tag{32}$$

$$G'(x_H, A_1, B_2) = 0, \qquad G'(x_L, A_1, B_2) = 0 \tag{33}$$

Note that

$$G''(x_H, A_1, B_2) < 0; \qquad G''(x_L, A_1, B_2) > 0 \tag{34}$$

B.1 Proof of equation (11) and (12)

Now, subtract (17) from (5) to see that G(x,n) satisfies the differential equation:

$$\frac{1}{2}\sigma^2 x^2 G''(x,n) + \alpha x G'(x,n) - \rho G(x,n) + x - C + \theta u(n) = 0 \tag{35}$$

Evaluating this at x_H and using (32), (33), and (35), we get:

$$x_H - C - \theta u(n) = \frac{1}{2}\sigma^2 x_H G''(x_H, n) + \alpha x_H G'(x_H, n) - \rho G(x_H, n) > -\rho K \tag{36}$$

than obtaining equation (11). In the peculiar case in which n is eqaul to zero then we get equation (9).

Similarly, we can get equation (12) and (10).

C Comparative statics

Although the equations defining the thresholds are highly nonlinear and do not have closed-form solutions, the total differentials corresponding to small changes in exogenous parameters are, as usual linear. This makes it relatively straightforward to obtain qualitative comparative statics results for at least certain parameters. We can show the effects of the investment cost K in detail and the effects of E and C are similar.

Now suppose that K changes by dK, and consider how the four endogenous variables A_1, B_2, x_L and x_H respond. Begin by differentiating the *value-matching* conditions (32) totally. Denote the partial derivatives of G by subscripts as usual, and write $G_A(x_H, A_1, B_2) = G_A(H)$, etc., for brevity. We obtain:

$$\begin{aligned} G_A(H)\,dA_1 + G_B(H)\,dB_2 &= dK \\ G_A(L)\,dA_1 + G_B(L)\,dB_2 &= 0 \end{aligned}$$

Note that the terms $G_P(H)dx_H$ and $G_P(L)dx_L$ have vanished because of the smooth-pasting conditions (33). Therefore, the general comparative static system in the four endogenous changes dA_1, dB_2, dx_L, and dx_H in fact separates in a simpler manner. First, we solve these two equations for the changes in the option value coefficients dA_1, dB_2. Then we can totally differentiate the smooth-pasting conditions to obtain the changes in the thresholds dx_H, dx_L.

Noting that $G_A(H) = x_H^{\beta_1}$, etc., the solution is

$$dA_1 = \frac{x_L^{\beta_2} dK}{\Delta}, \qquad dB_1 = \frac{x_L^{\beta_1} dK}{\Delta}$$

where

$$\Delta = x_H^{\beta_1} x_L^{\beta_2} - x_H^{\beta_2} x_L^{\beta_1}$$

which is positive because $x_H > x_L$ and $\beta_1 > 0 > \beta_2$.

Now differentiate the *smooth-pasting* condition at x_H in (33) to write

$$G''(H)\,dx_H + G'_A(H)\,dA_1 + G'_B dB_2 = 0$$

which gives

$$G''(H)\,dx_H = -\frac{\left[\beta_1 x_H^{\beta_1-1} x_L^{\beta_2} - \beta_2 x_H^{\beta_2-1} x_L^{\beta_1}\right] dK}{\Delta}$$

Since $G(x)$ is concave at x_H, $G''(H)$ is negative and then $dx_H > 0$ when $dK > 0$. The investment threshold rises with the investment cost, as we should expect. Similarly, x_L falls as E rises.

Similarly, the lower smooth-pasting condition gives:

$$G''(L)\,dx_L = -\frac{(\beta_1 - \beta_2)\,x_L^{\beta_1+\beta_2-1}dK}{\Delta}$$

Since $G''(L) > 0$, we have $dx_L < 0$ when $dK > 0$.

D Policy Uncertainty

Dixit and Pindyck (1993) affirm that "*policy uncertainty is not likely to be well captured by a Brownian motion process; it is more likely to be a Poisson jump*". Therefore, our model changes in the following manner: if θ follows a jump process, we write this by analogy with the notation for Brownian motion as:

$$d\theta = \gamma\theta dt + \theta dq \tag{37}$$

where dq is the increment of a Poisson process with mean arrival rate γ, and dq is independent from dw. [so that $E(dzdq) = 0$]. We will assume that if an "event" occurs, q falls by some fixed percentage with probability 1. By the brownian motion study in (1), we know that,

$$\begin{aligned} E(dw)^2 &= dt \\ (dx)^2 &= \sigma^2 x^2 dt \end{aligned}$$

Let us denote (Dixit and Pindyck , 1993, p.85) a Poisson process by analogy with the weiner process. In other words, let dq be equal to 0 with probability $1 - \varpi dt$ and equal to $-\phi$ with probability ϖdt, so that

$$E(d\theta) = \gamma\theta dt - \theta\phi\varpi dt$$

If the two variables x and θ follow respectively a geometric brownian motion and a jump process we can use *Itô's Lemma* to calculate dV, writing (Dixit and Pindyck, 1993, p.209):

$$dV(x,\theta,n) = \frac{\partial V}{\partial t}dt + \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial \theta}d\theta + \frac{1}{2}\frac{\partial^2 V}{\partial x^2}(dx)^2 \tag{38}$$

And substituting (1), (37), into (38), dividing all by dt and rearranging we can obtain the expected value of dV:

$$E(dV) = \frac{\partial V}{\partial x}\alpha x + \frac{\partial V}{\partial \theta}\theta\gamma + \frac{1}{2}\frac{\partial^2 V}{\partial x^2}\sigma^2 x^2 + \varpi\left\{V\left[(1-\phi)x\right] - V(x)\right\} \tag{39}$$

And now using (39) in real option framework we have:

$$\rho V = \frac{1}{2}\frac{\partial^2 V}{\partial x^2}\sigma^2 x^2 + \frac{\partial V}{\partial x}\alpha x + \frac{\partial V}{\partial \theta}\theta\gamma + \varpi\left\{V\left[(1-\phi)x\right] - V(x)\right\} + \left[x + \theta u(n)\right]$$
$$\frac{1}{2}\frac{\partial^2 V}{\partial x^2}\sigma^2 x^2 + \frac{\partial V}{\partial x}\alpha x + \frac{\partial V}{\partial \theta}\theta\gamma - (\rho+\varpi)V + \varpi V\left[(1-\phi)x\right] + \left[x + \theta u(n)\right] \tag{40}$$

To solve (40), we can use a simplification from Dixit and Pindyck (p. 210):

$$\begin{aligned} V(x,\theta,n) &= \theta u f\left(\frac{x}{\theta u(n)}\right) = \theta u f(s) \\ \frac{\partial V}{\partial x} &= f'(s) \\ \frac{\partial^2 V}{\partial x^2} &= \frac{f''(s)}{\theta u} \\ \frac{\partial V}{\partial \theta} &= uf(s) - usf'(s) \end{aligned} \tag{41}$$

Substituting (41) into (40) we obtain:

$$\frac{1}{2}\frac{f''(s)}{\theta u}\sigma^2 x^2 + f'(s)\alpha x + \left[uf(s) - usf'(s)\right]\theta\gamma - (\rho+\varpi)\theta u f(s) + \tag{42}$$
$$+\varpi\theta u f\left[(1-\phi)s\right] + \left[x + \theta u(n)\right] \tag{43}$$

rearranging and dividing all by θu

$$\frac{1}{2}f''(s)\sigma^2 s^2 + f'(s)s\left[\alpha-\gamma\right] - f(s)\left[\varpi+\rho-\gamma\right] + \varpi f\left[(1-\phi)s\right] + s + 1 \tag{44}$$

Now, we can look at the general solution as the sum of a solution of the homogeneous equation plus a particular solution of the inhomogeneous equation. The first step is the analysis of the homogeneous equation:

$$\frac{1}{2}f''(s)\sigma^2 s^2 + f'(s)s\left[\alpha-\gamma\right] - f(s)\left[\varpi+\rho-\gamma\right] + \varpi f\left[(1-\phi)s\right] \tag{45}$$

The solution of (45) is again $f(s) = As^{\beta_1}$, but now is the positive solution to a slightly more complicated non-linear equation:

$$\frac{1}{2}\beta(\beta-1)\sigma^2 + \beta\left[\alpha-\gamma\right] - \left[\varpi+\rho-\gamma\right] + \varpi(1-\phi)^{\beta} = 0 \tag{46}$$

The value of β that satisfies (46) and $f(0) = 0$ can be found numerically. The general solution of (44) appears to be the following:

$$f(s) = As^{\beta_1} + \pi(s) \tag{47}$$

where $\pi(s)$ is a particular solution of (44).

It is possible to demonstrate that the study done until now, could be simplified by reducing our analysis of the sum of two variables following stochastic processes to the analysis of a combined brownian motion and a jump process as shown in Dixit and Pindyck (pp. 167-173). In the same way the drift of the jump process can be included in the drift of the brownian motion or erased. In the following analysis let us for simplicity set $\gamma = 0$ and considering that:

$$f_1 = \frac{(1-\phi)\,s}{\rho-\alpha}$$

the solution of (44) is:

$$\begin{aligned}
\pi\,(s) &= \frac{\varpi\,(1-\phi)\,s}{(\rho-\alpha)\,(\rho-\alpha+\varpi)} + \frac{s}{(\rho-\alpha+\lambda)} + \frac{1}{(\rho+\varpi)} && (48)\\
&= \frac{s}{(\rho-\alpha+\varpi)} \cdot \left[\frac{\varpi\,(1-\phi)}{(\rho-\alpha)} + 1\right] + \frac{1}{(\rho+\varpi)} && (49)
\end{aligned}$$

Thus the general solution is:

$$f(s) = As^{\beta_1} + \frac{s}{(\rho-\alpha+\varpi)} \left[\frac{\varpi\,(1-\phi)}{(\rho-\alpha)} + 1\right] + \frac{1}{(\rho+\varpi)} \tag{50}$$

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